

Lesson 1 (B): Gyromagnetic ratio Larmor frequency

Aims

- Learning the following topics:
- Spin quantum number
- Number of energy states
- Hydrogen proton and MRI
- Protons' spin
- Protons' precession
- Larmor equation
- Gyromagnetic ratio

Spin quantum number

- According to quantum theory, each atomic nuclei has specific energy levels related to a property called spin quantum number S .
- For example, the hydrogen nucleus (a single proton) has a spin quantum number S of $\frac{1}{2}$.
- $S(1H) = \frac{1}{2}$

Number of energy states

- The number of energy states of a nucleus is determined by the formula:
- # of energy states = $2S + 1$
- For a proton with a spin $S = \frac{1}{2}$:
- # of energy states = $2 \left(\frac{1}{2}\right) + 1 = 1 + 1 = 2$
- Therefore, a hydrogen proton has two energy states denoted as $-\frac{1}{2}$ and $+\frac{1}{2}$.

Energy states of the hydrogen proton

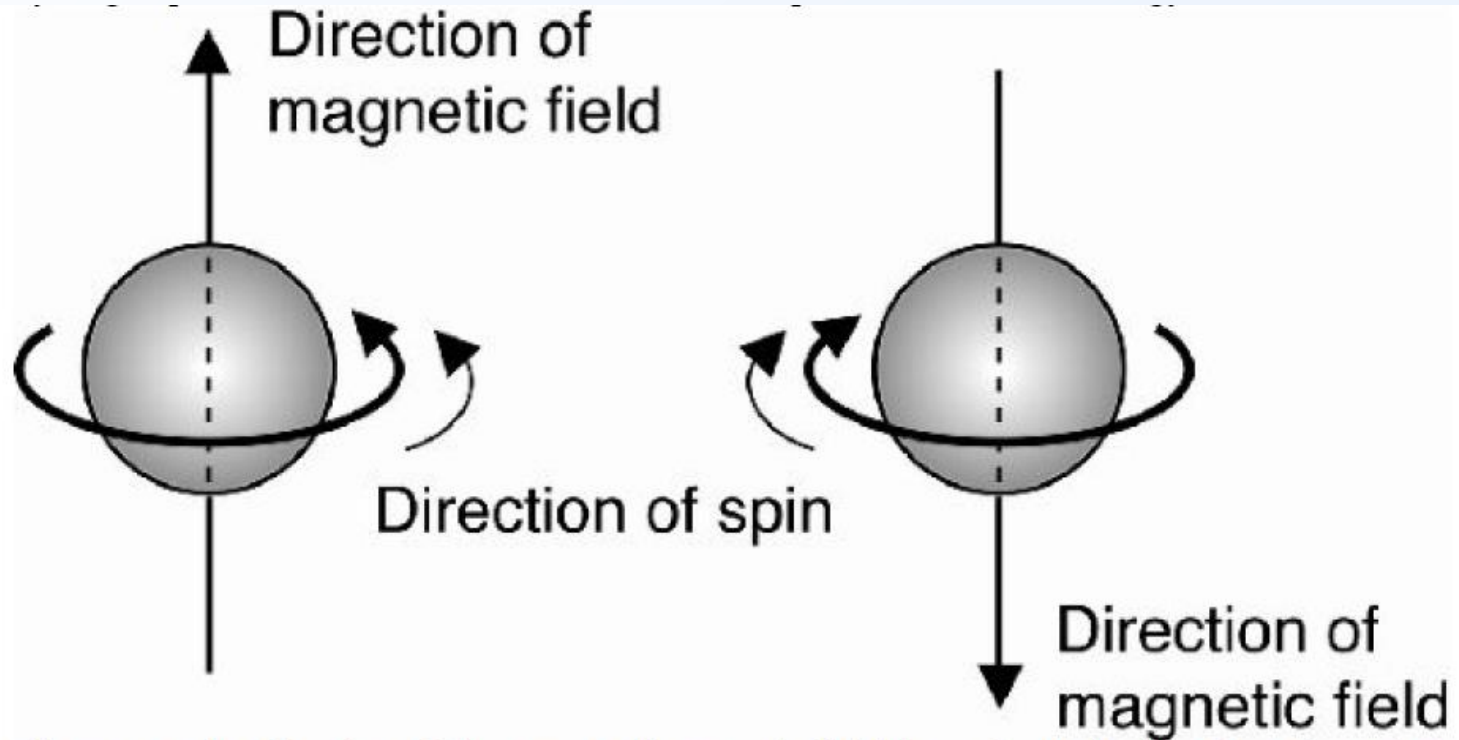


Figure 2-6 The direction of the generated magnetic field depends on the direction of rotation of the spinning protons.

Spin up: $+1/2$
(lower energy state)

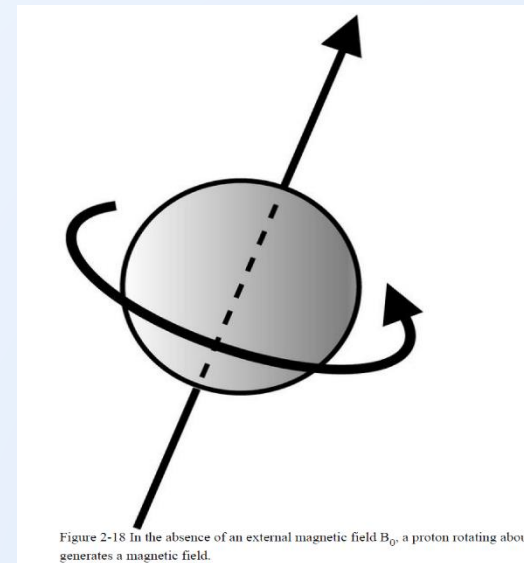
Spin down: $-1/2$
(higher energy state)

Hydrogen protons for MRI

- We find hydrogen protons (^1H), for example, in water (H_2O) and fat ($-\text{CH}_2-$).

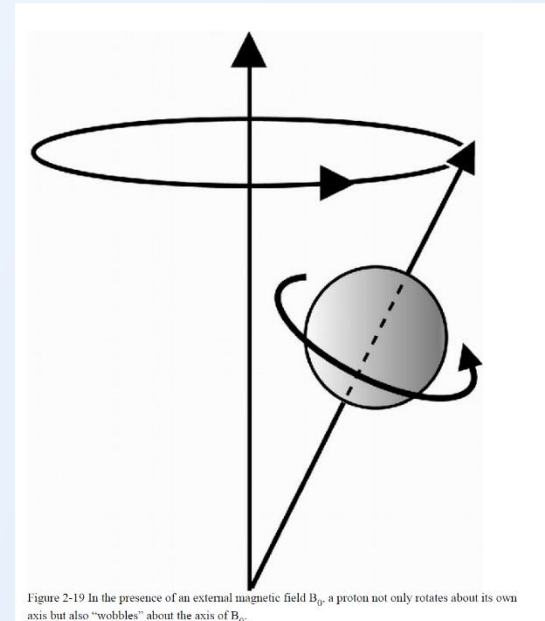
Proton spin

- In the absence of an external magnetic field (B_0), single proton spinning about its axis and generates its own small magnetic field.

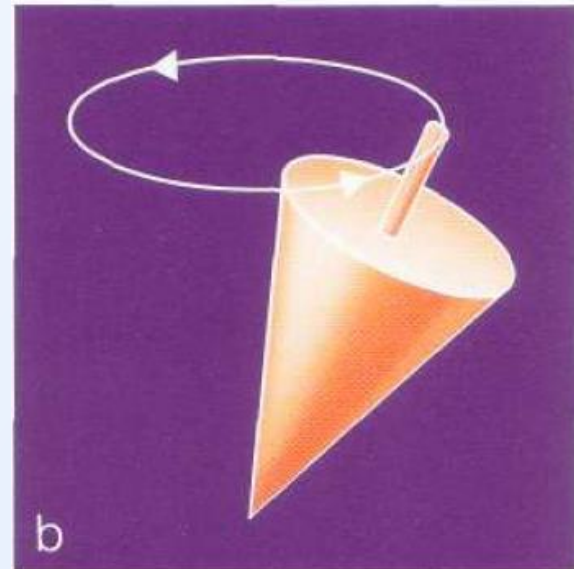
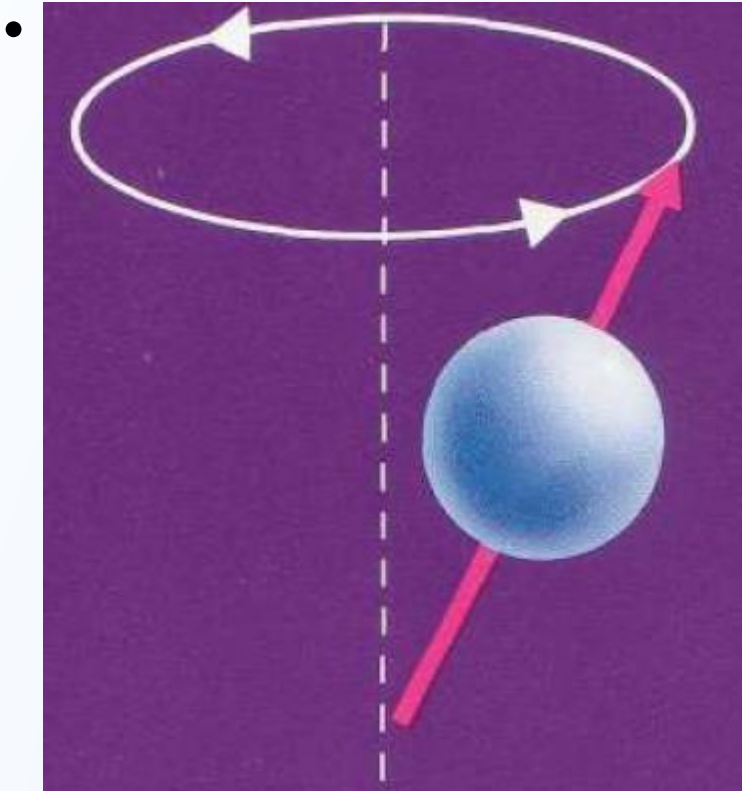


Proton precession

- When a proton is placed in a large magnetic field (B_0), not only spins about its own axis, but also rotates or “precesses” about the axis of the external magnetic field (B_0).

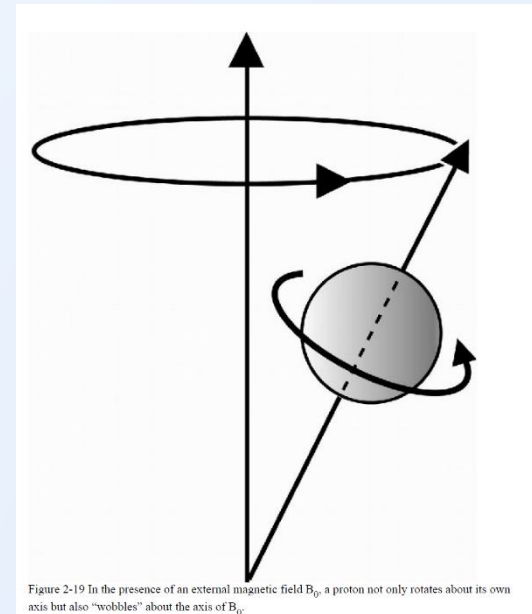


Precession



Question

- Which one is faster? Spin or precession?



Larmor Equation

- The rate at which the proton precesses around the external magnetic field is given by an equation called the Larmor equation.

Larmor equation

$$\omega_0 = \gamma B_0$$

Angular precessional
frequency of proton
(Larmor frequency)
(MHz)

gyromagnetic
Ratio
(MHz/T)

strength of the
external magnetic
field (T)

The gyromagnetic ratio

- The gyromagnetic ratio γ is a constant that is fixed for each nucleus. For hydrogen proton, $\gamma (\text{H}) = 42.6 \text{ MHz/Tesla}$.

Larmor frequency

- As the external magnetic field strength increases, the precessional frequency of the hydrogen proton
.....
- increases.
- $\omega_0 = \gamma B_0$
- 42.6 MHz = 42.6 * 1
- 64 MHz \approx 42.6 * 1.5
- 128 MHz \approx 42.6 * 3

Summary

- Spin quantum number
- Number of energy states
- Hydrogen proton and MRI
- Protons' spin
- Protons' precession
- Larmor equation
- Gyromagnetic ratio

Reference

- Hashemi RH and Brandy WG. MRI the Basics, Second Edition.